THE PERFECT MAGIC SQUARES FOR $\mp$1909.*

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In an article on Magic Squares in *Our Schoolday Visitor, Mathematical Almanac and Annual,* 1871, Judge Scott of the Ohio Supreme Court divides them into, simple, perfect, complex, compound, triple, quadruple and quintuple magic squares. I will add one more — potential magic squares, or those existing in possibility, not in reality — being too great to construct.

There can be magic squares — and some very interesting ones — that involve both positive and negative numbers, that respond to general laws for their development. The magic squares for the years $\mp$1909 all belong to this class.

Let y=the year; then we have by Zerr's formula†,

$$2y=n[2x\pm(n^2-1)];$$

from which

$$x=[2y\mp(n^3-n)]/2n.$$

We know that $1909=23\times83$, the product of two primes, from which

I. When $y=1909$, and $n=23$, we have for the least value, $x=-181$; and for the greatest value, $x=347$.

II. When $y=1909$, and $n=83$, we have for the least value, $x=-3421$; and for the greatest value, $x=3457$.

III. When $y=-1909$, and $n=23$, we have for the least value, $x=-347$; and for the greatest value, $x=181$.

IV. When $y=-1909$, and $n=83$, we have for the least value, $x=-3467$; and for the greatest value, $x=3421$.

Judge Scott also claims that if n be a prime number, not less than 5, we may in every such case construct a great variety of perfect magic squares, and gives the general rule for their construction; and concludes with the remark, that the number of such magic squares may be expressed by the formula, $n^2(n-1)^2(n-3)(n-4)$.

The number of systems of magic squares for the years, -1909 and $+1909$ is limited to I, II, III, and IV, given above, and to the variations of each.

By the formula we have for

I. 97,293,680 magic squares for 1909;

II. 292,752,739,520 magic squares for 1909;

III. 97,293,680 magic squares for -1909; and

IV. 292,752,739,520 magic squares for -1909; or a

Total of 585,700,066,400 magic squares for the year $+1909$ and -1909.

*In the January, 1909, issue of the MONTHLY, magic squares for 1909 were requested. This article is in response to that request.

†Vol. XVI, No. 1, page 2, of the MONTHLY.

We will now develop a perfect magic square for −1909, when $n=23$. The year, −1909, may be interpreted as before Christ. In each series from $-x$ to $+x$, zero must occupy one cell in all of these magic squares.

There is no magic square for positive integers alone for 1909.

The following perfect magic square for −1909 begins with −347 and ends with +181; but the corresponding magic square for +1909 would begin with −181 and end with +347.

MAGIC SQUARE FOR THE YEAR, −1909.

-70	-45	-20	5	30	55	80	105	130	155	180	-347	-322	-297	-272	-247	-222	-197	-172	-147	-122	-97	-72
-46	-21	4	29	54	79	104	129	154	179	-325	-323	-298	-273	-248	-223	-198	-173	-148	-123	-98	-73	-71
-22	3	28	53	78	103	128	153	178	-326	-324	-299	-274	-249	-224	-199	-174	-149	-124	-99	-74	-49	-47
2	27	52	77	102	127	152	177	-327	-302	-300	-275	-250	-225	-200	-175	-150	-125	-100	-75	-50	-48	-23
26	51	76	101	126	151	176	-328	-303	-301	-276	-251	-226	-201	-176	-151	-126	-101	-76	-51	-26	-24	1
50	75	100	125	150	175	-329	-304	-279	-277	-252	-227	-202	-177	-152	-127	-102	-77	-52	-27	-25	0	25
74	99	124	149	174	-330	-305	-280	-278	-253	-228	-203	-178	-153	-128	-103	-78	-53	-28	-3	-1	24	49
98	123	148	173	-331	-306	-281	-256	-254	-229	-204	-179	-154	-129	-104	-79	-54	-29	-4	-2	23	48	73
122	147	172	-332	-307	-282	-257	-255	-230	-205	-180	-155	-130	-105	-80	-55	-30	-5	20	22	47	72	97
146	171	-333	-308	-283	-258	-233	-231	-206	-181	-156	-131	-106	-81	-56	-31	-6	19	21	46	71	96	121
170	-334	-309	-284	-259	-234	-232	-207	-182	-157	-132	-107	-82	-57	-32	-7	18	43	45	70	95	120	145
-335	-310	-285	-260	-235	-210	-208	-183	-158	-133	-108	-83	-58	-33	-8	17	42	44	69	94	119	144	169
-311	-286	-261	-236	-211	-209	-184	-159	-134	-109	-84	-59	-34	-9	16	41	66	68	93	118	143	168	-336
-287	-262	-237	-212	-187	-185	-160	-135	-110	-85	-60	-35	-10	15	40	65	67	92	117	142	167	-337	-312
-263	-238	-213	-188	-186	-161	-136	-111	-86	-61	-36	-11	14	39	64	89	91	116	141	166	-338	-313	-288
-239	-214	-189	-164	-162	-137	-112	-87	-62	-37	-12	13	38	63	88	90	115	140	165	-339	-314	-289	-264
-215	-190	-165	-163	-138	-113	-88	-63	-38	-13	12	37	62	87	112	114	139	164	-340	-315	-290	-265	-240
-191	-166	-141	-139	-114	-89	-64	-39	-14	11	36	61	86	111	113	138	163	-341	-316	-291	-266	-241	-216
-167	-142	-140	-115	-90	-65	-40	-15	10	35	60	85	110	135	137	162	-342	-317	-292	-267	-242	-217	-192
-143	-118	-116	-91	-66	-41	-16	9	34	59	84	109	134	136	161	-343	-318	-293	-268	-243	-218	-193	-168
-119	-117	-92	-67	-42	-17	8	33	58	83	108	133	158	160	-344	-319	-294	-269	-244	-219	-194	-169	-144
-95	-93	-68	-43	-18	7	32	57	82	107	132	157	159	-345	-320	-295	-270	-245	-220	-195	-170	-145	-120
-94	-69	-44	-19	6	31	56	81	106	131	156	181	-346	-321	-296	-271	-246	-221	-196	-171	-146	-121	-96